Geant4 Electromagnetic Physics

V.Ivanchenko,
thanks to M.Maire

- Physics categories
- Electromagnetic physics
- Standard EM package
- PhysicsList and Cuts
- Lowenergy EM package
- Particle EM interactions with matter

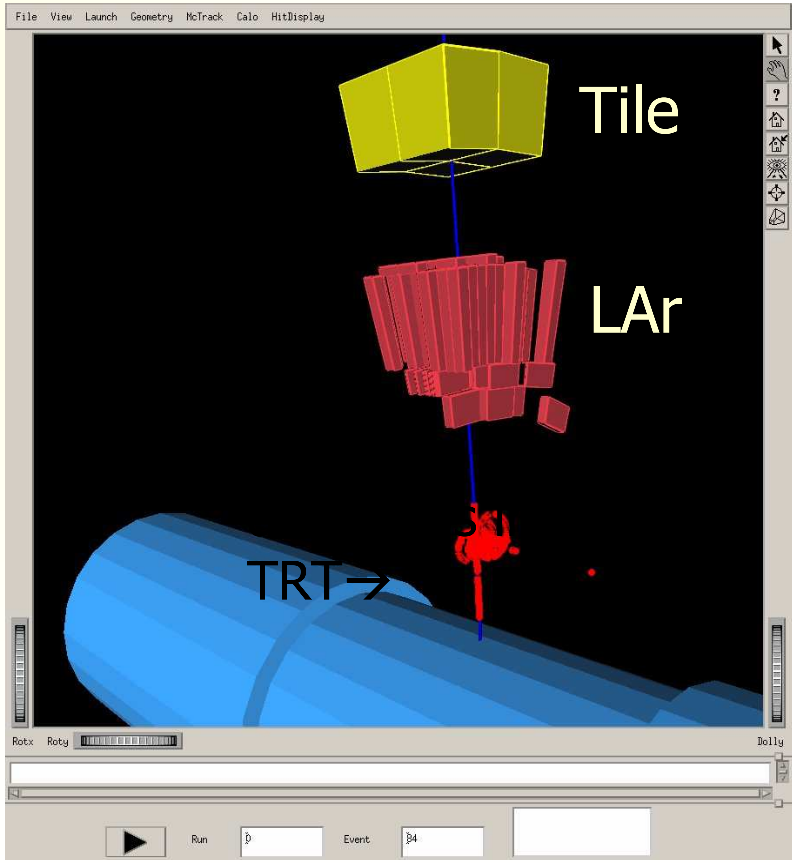
Geant4 physics processes

- Physics is described via abstract interface called ***process*** associated with ***particles***
- ***Process*** provides *Interaction Lenths*, *StepLimits*, and *DoIt* methods
- ***Process*** active *AlongStep, PostStep, AtRest*
- Distinction between process and model – one process may includes many models
- Generation of final state is well separated (for many processes is independed) from the access and use of cross sections and from tracking

Geant4 physics categories

- There are following categories:
  - Decay
  - Electromagnetic
  - Hadronic
  - Optical
  - Transportation
  - Parameterisations

ATLAS



Electromagnetic Physics

- Processes of gamma, electron, and positron interactions with media was traditionally called ***"Electromagnetic Processes" (EM)***
- Hadron interaction with atomic electrons are also EM
- Hadron photo- and electro- production are simulated in framework of G4 hadronic physics

EM packages

- ***Standard*** – basic set of processes for HEP
- ***Muons*** – basic set of muon processes for HEP
- ***Xrays*** – xray and optical proton production
- ***Lowenergy*** – alternative set of processes with low energy extension of gamma, electron, and hadron EM physics
- ***Highenergy*** – EM processes important above 100 GeV
- ***Optical*** – Optical photon interaction
- ***Utils – common classes for other EM packages:***
  - ***Interfaces***
  - ***Energy loss and range table builders***
  - ***Useful utilities***

Standard EM Physics

- The projectile is assumed to have the energy $E_{kin} > 1keV$
- The atomic electrons are quasi-free – their binding energies neglected (except some corrections at low energies)
- The atomic nucleus are fixed – no recoil
- The matter is described as homogeneous, isotropic, amorphous

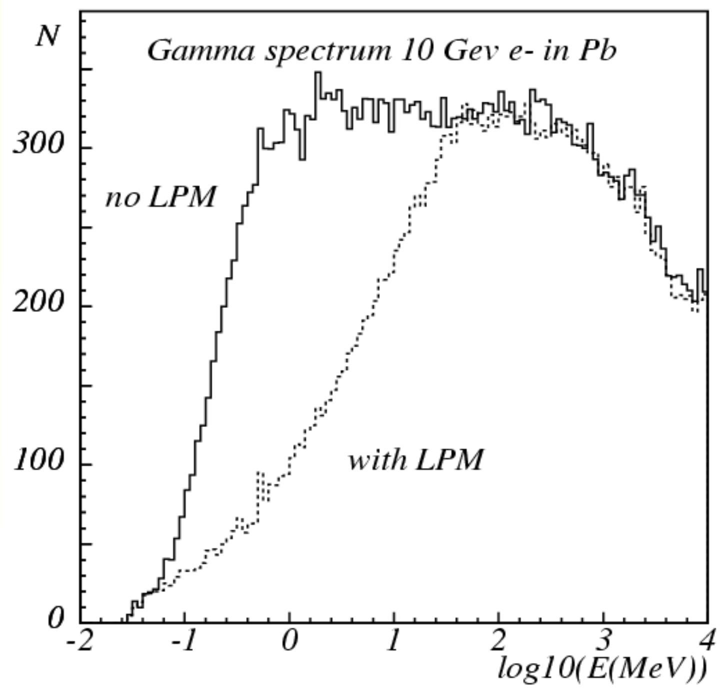
Standard EM Processes

- Gamma
  - Photo-electric effect
  - Compton scattering
  - e^+e^- pair production
  - $\mu^+\mu^-$ pair production
- Electron and positron
  - Ionization
  - Bremsstrahlung
  - Positron annihilation
- Muons
  - Ionization
  - Bremsstrahlung
  - e^+e^- pair production
- Hadrons
  - Ionization
- Ions
  - Ionization
- Multiple scattering

Standard EM Physics

- Standard G4 physics was based on G3 knowledge/experience
- Review of G3 models have been done
- More precise theories were used if possible/ necessary
- Extension to higher energies in progress

Landau-Pomeranchuk-Migdal Effect for bremsstrahlung

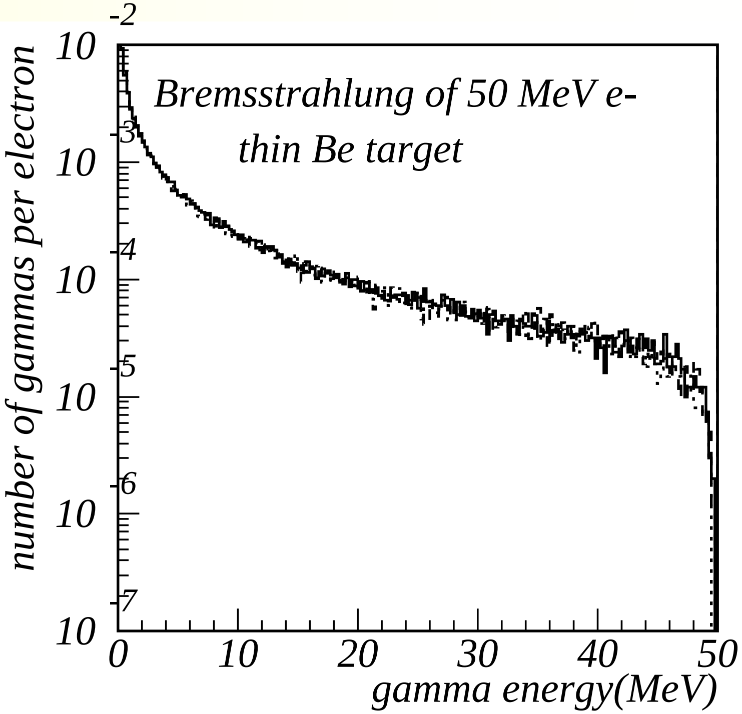


Standard EM Physics

- Standard package of EM interactions was created for HEP applications
- It is well adequate for instrumental studies, space and medical applications
- Examples of different usage of the Standard package:
  - $G4INSTALL/examples/novice
  - $G4INSTALL/examples/extended/electromagnetic
  - $G4INSTALL/examples/extended/medical
- Examples of PhysicsList in the directory $G4INSTALL/physics_list/electromagnetic

Energy Cuts for EM Physics

- Energy spectrum of δ-electrons ~ $1/T^2$
- Energy spectrum of Bremsstahlung ~ $1/\omega$
- Huge number of low energy e- and gammas cannot be tracked efficiently by any Monte Carlo
- Cuts should be used



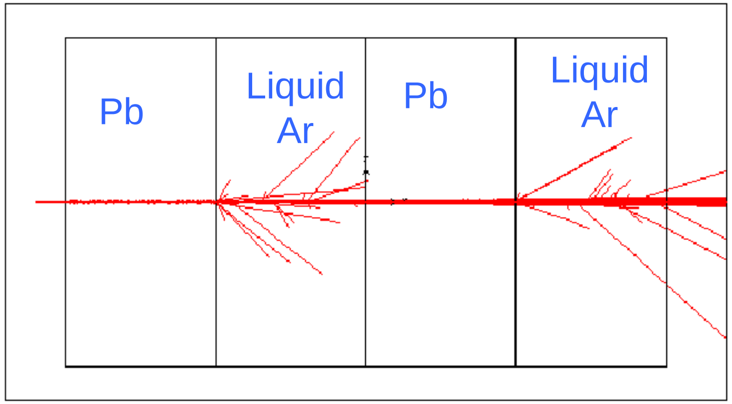
Geant4 cuts

- For a typical process G4Ionisation production threshold T_c subdivides continues and discrete part of energy loss:
- Energy loss $$\frac{dE}{dx} = n \int_0^{Tc} t \frac{d\sigma(t)}{dt} dt$$
- δ-electron production $$\sigma = \int_{Tc}^{T\max} \frac{d\sigma}{dt} dt$$
- By default energy is deposited at the step
- Energy loss can be used optionally for generation of δ-electrons under the threshold (subcutoff) and for fluorescence and Auger–electrons emission

Effect of production thresholds

Geant 4

500 MeV incident proton



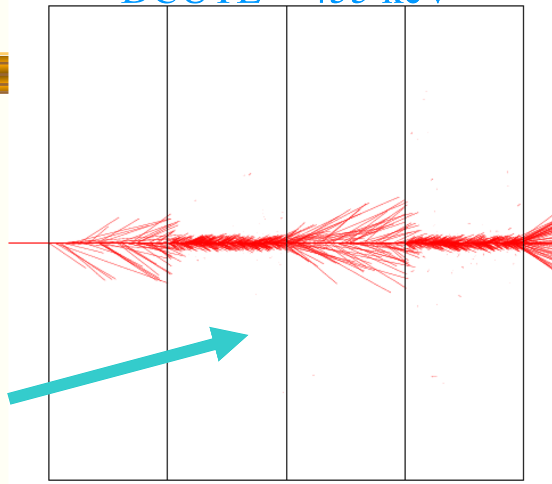
Range threshold: 1.5 mm

455 keV electron energy in liquid Ar

2 MeV electron energy in Pb

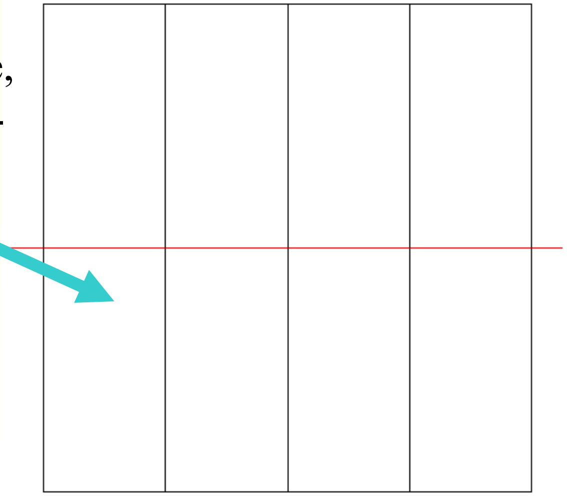
In Geant3

DCUTE = 455 keV



one must set the cut for delta-rays (DCUTE) either to the Liquid Argon value, thus producing many small unnecessary δ-rays in Pb,

or to the Pb value, thus killing the δ-rays production everywhere



DCUTE = 2 MeV

Remarks about Geant4 cuts

- The use of production threshold is mandatory only for Standard ionization and bremsstahlung
- Other processes can use or ignore G4 cuts
- Alternative mechanism is UserLimits, which can be defined in a given G4LogicalVolume:
  - Maximum step size
  - Maximum track length
  - Maximum track time
  - Minimun kinetic energy
  - Minimum range

PhysicsList

- It is one of the « mandatory user classes »;
  - Defined in source/run
- Defines the three pure virtual methods:
  - ConstructParticle()
  - ConstructProcesse()
  - SetCuts()
- Concrete PhysicsList needs to inherit from G4VUserPhysicsList or G4VModularPhysicsList
- For interactivity G4UserPhysicsListMessenger can be used to handle PhysicsList parameters

Example: Gamma processes

- Discrete processes - only PostStep actions;
  - Use function AddDiscreteProcess;
  - **pmanager** is the G4ProcessManager of the gamma;
  - Assume the transportation has been set by AddTransportation;
- Code sample:

```
// Construct processes for gamma:
pmanager->AddDiscreteProcess(new G4GammaConversion());
pmanager->AddDiscreteProcess(new G4ComptonScattering());
pmanager->AddDiscreteProcess(new G4PhotoElectricEffect());
```

Example: electron and positron

Main interface with definition of the process order:

G4ProcessManager::AddProcess(G4VProcess*, int orderAtRest,
int orderAlongStep, int orderPostStep);

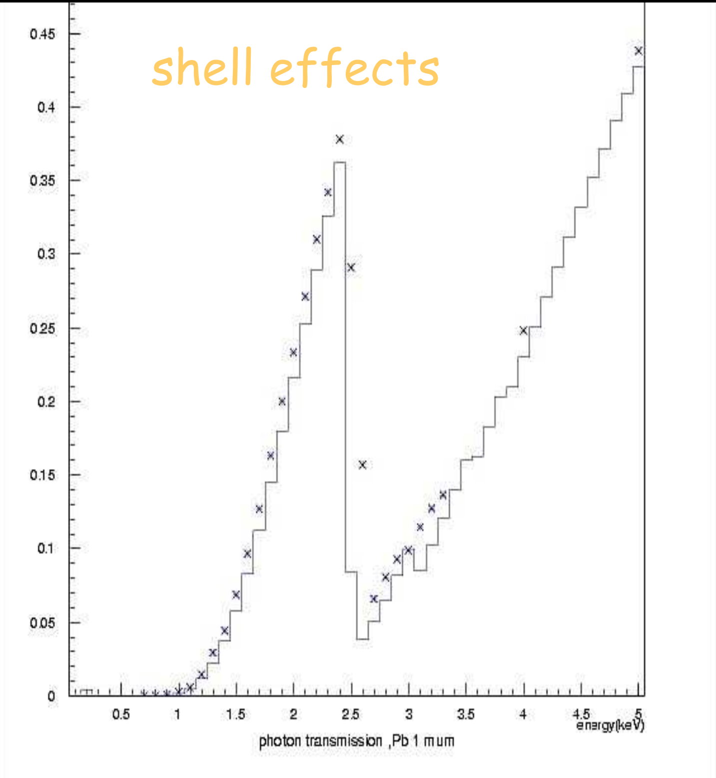
NOTE: if (order < 0) – process inactive; else – the order of DoIt method;
inverse order of GetInteractionLength

```
// add processes for e-
G4ProcessManager* pmanager = G4Electron::Electron()->GetProcessManager();
pmanager->AddProcess (new G4MultipleScattering,  -1,  1,  1 );
pmanager->AddProcess (new G4eIonisation,          -1,  2,  2 );
pmanager->AddProcess (new G4eBremsstrahlung,      -1,  3,  3 );
// add processes for e+
pmanager = G4Positron::Positron()->GetProcessManager();
pmanager->AddProcess (new G4MultipleScattering,  -1,  1,  1 );
pmanager->AddProcess (new G4eIonisation,          -1,  2,  2 );
pmanager->AddProcess (new G4eBremsstrahlung,      -1,  3,  3 );
pmanager->AddProcess (new G4eplusAnnihilation,     1, -1,  4 );
```

Geant4 low energy EM physics

- Validity down to **250 eV**
  - 250 eV is a “suggested” lower limit
  - data libraries down to 10 eV
  - 1 < Z < 100
- Exploit **evaluated data libraries** (from LLNL):
  - EADL (Evaluated Atomic Data Library)
  - EEDL (Evaluated Electron Data Library)
  - EPDL97 (Evaluated Photon Data Library)

Photon transmission, 1μm Pb



Geant4 low energy EM physics

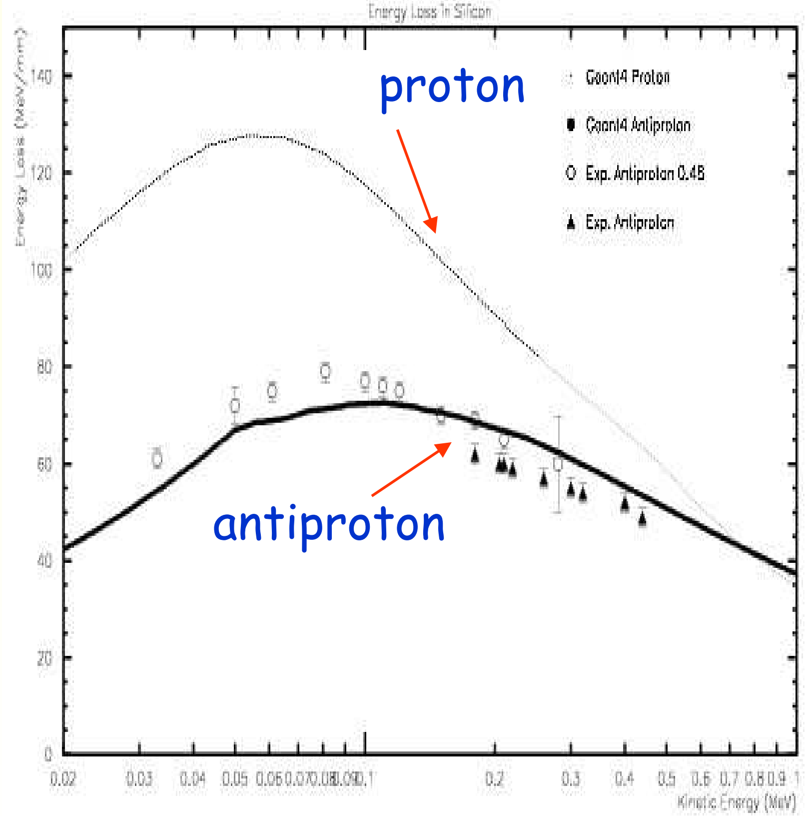
- Compton scattering
- Polarised Compton
- Rayleigh scattering
- Photoelectric effect
- Pair production
- Bremsstrahlung
- Electron ionisation
- Hadron ionisation
- Atomic relaxation
- Set of Penelope models (new)

- It is relatively new package
- Development is driven by requirements which come from medicine and space research
- There are also users in HEP instrumentation
- There is a long list of new development to be implemented including physics in 10-250 eV energy range

Geant4 low energy EM physics

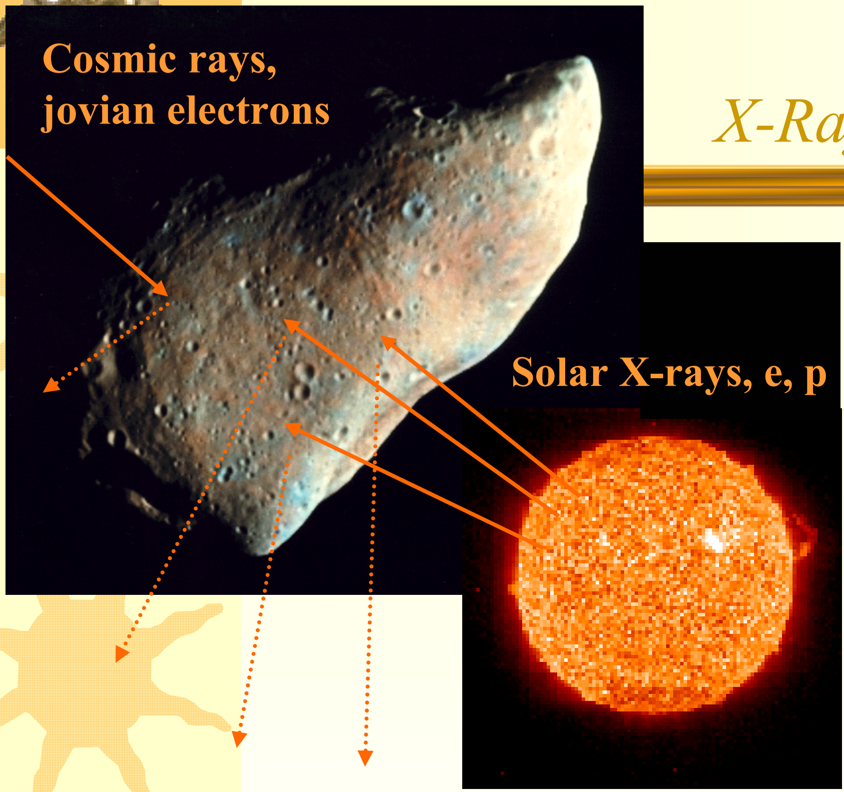
- Ionization is different for particles and antiparticles (Barkas effect)
- Ionization at low energy depends on molecular shell structure
- Chemical formula can be assign to the material – will be effective for heights of the Bragg peak of ionization

Energy loss in Silicon



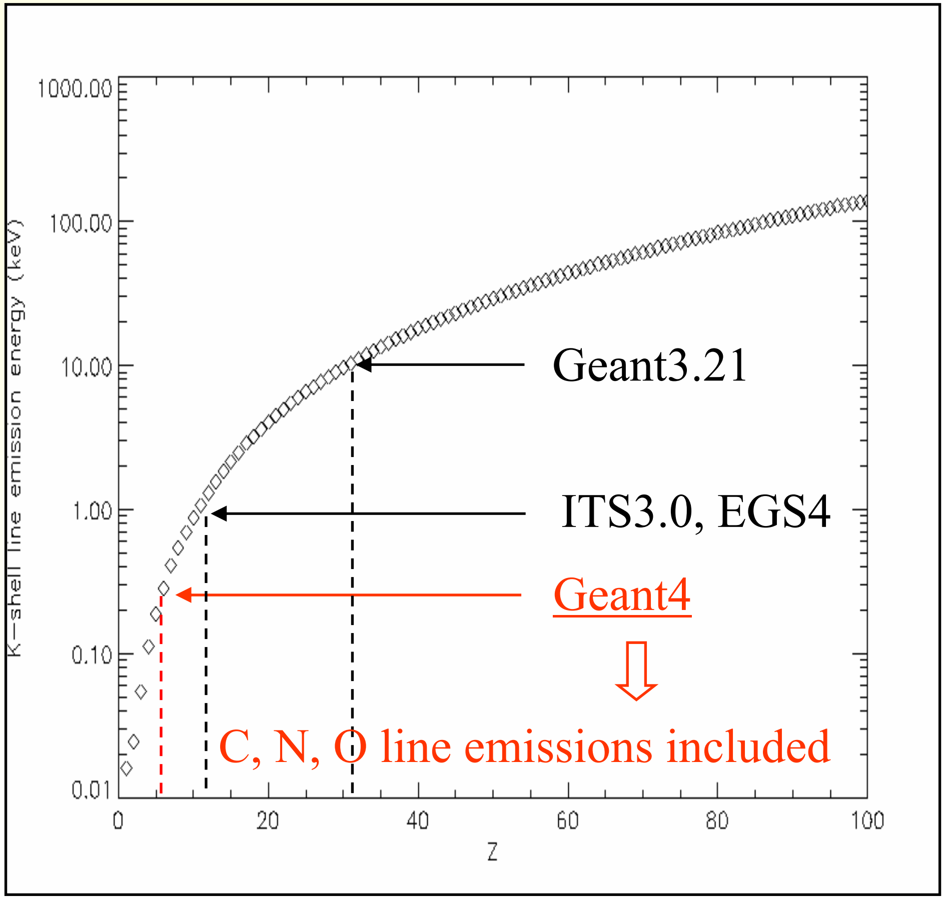
Geant4 low energy EM physics (contingency M.G.Pia)

X-Ray Surveys of Solar System Bodies



Courtesy SOHO EIT

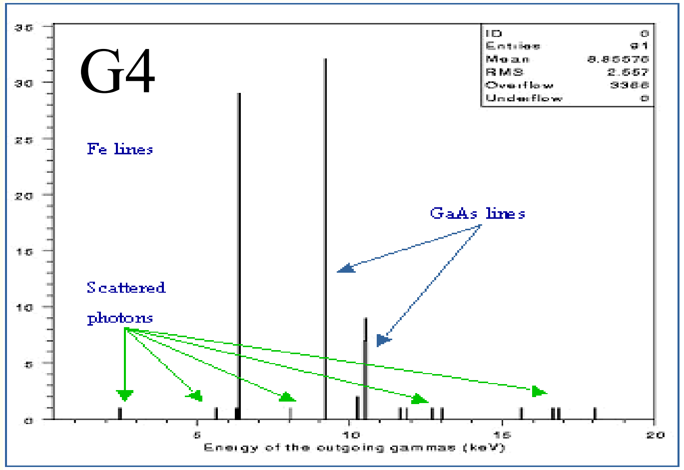
Induced X-ray line emission:
indicator of target composition
(~100 μm surface layer)

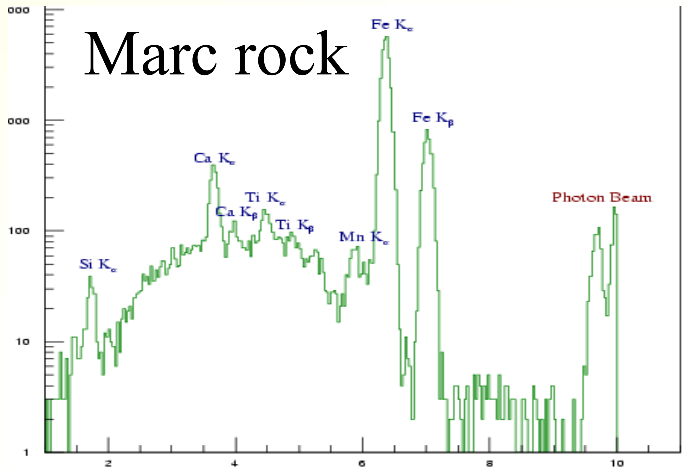


Geant4 low energy EM physics

(contingency M.G.Pia)

- Atomic relaxations are implement for ionization processes and photoelectric effect
- Cross sections of shell ionization are used
- Fluorescence and Auger electrons are produced



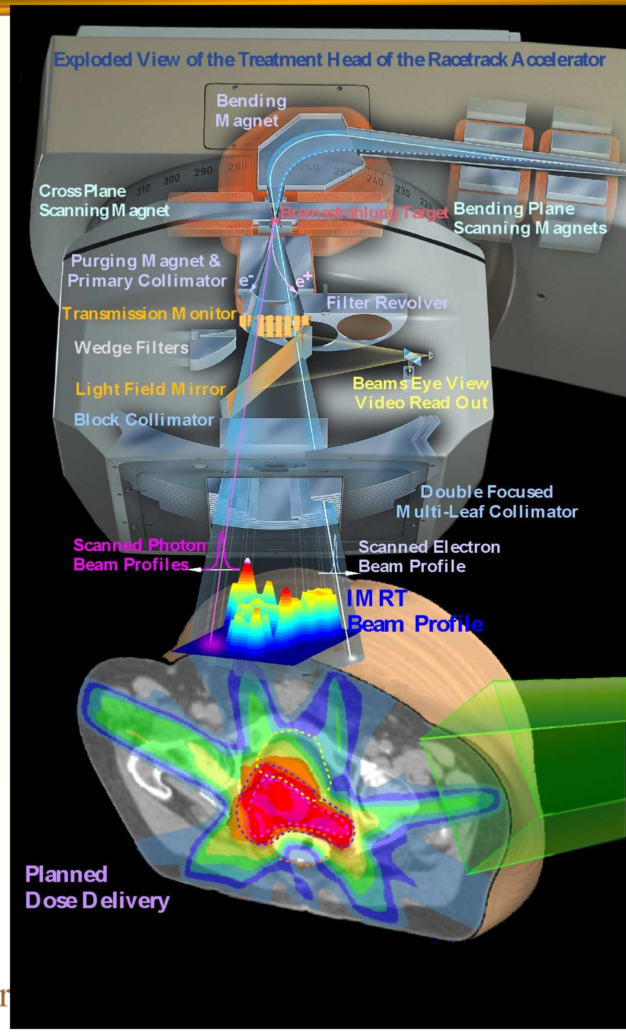


Geant4 low energy EM physics

- ✶ To use G4 lowenergy package user has to substitute standard process in the PhysicsList by corresponding lowenergy:
  - G4hIonisation → G4hLowEnergyIonisation
  - G4eIonisation → G4LowEnergyIonisation
  - ……
- ✶ The environment variable G4LEDATA should be defined

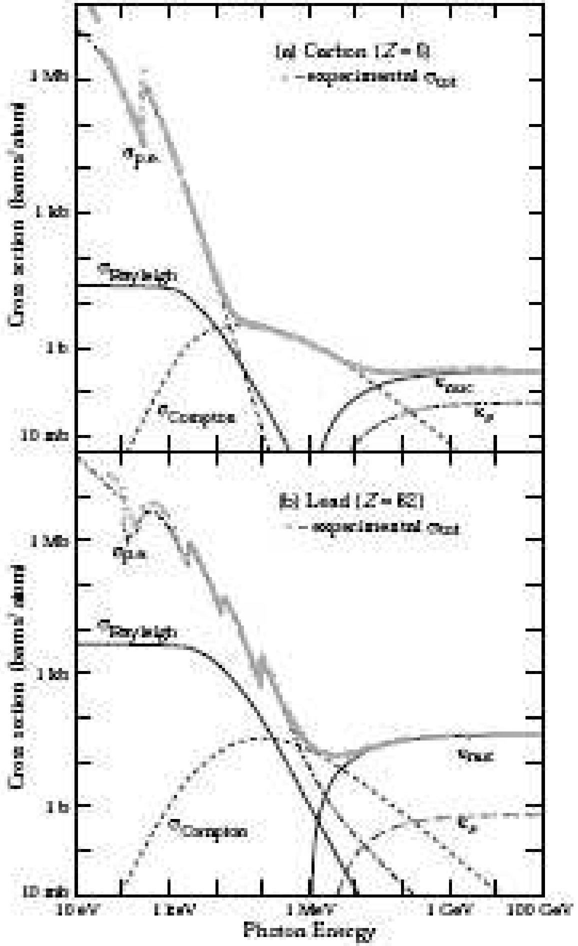
Particle EM interactions with matter for the Standard package

- Gamma interaction
- Electron and positron interactions
- Heavy charged particles interactions
- All interactions are needed to understand the details of radiation treatment



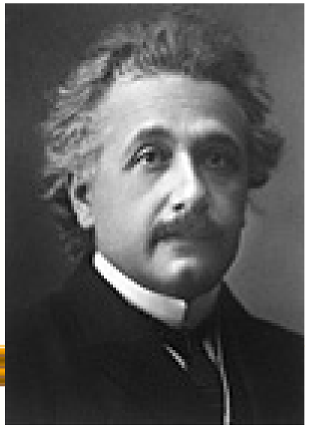
Gamma Cross Sections (PDG)

- Atomic photoelectric effect
- Coherent scattering (Rayleigh)
- Incoherent scattering (Compton effect)
- Pair production, nuclear field
- Pair production, electron field
- Data from NIST

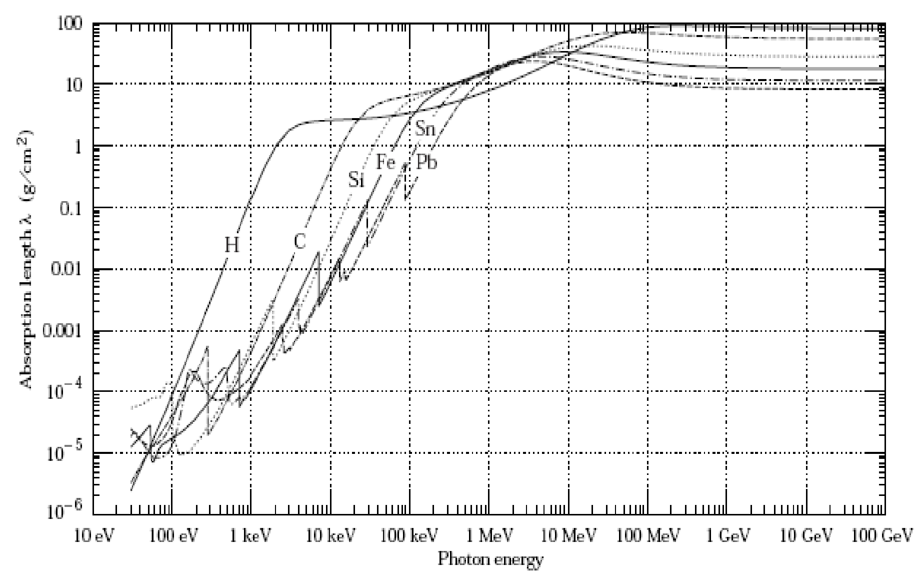


http://physics.nist.gov/PhysRefData

Photoelectric Effect



- Reaction
  - $\gamma + A \rightarrow e^{-} + A^{*}$
- Discontinuity
  - grow up of cross section when $E_\gamma \rightarrow E_{bound}$
- K-shell dominates above
- Parameterization:



$$\sigma(Z, E_\gamma) = \sum_{i=1,4} \frac{c_i(Z, E_\gamma)}{E_\gamma^i}$$

http://physics.nist.gov/PhysRefData

http://www-cxro.lbl.gov/optical_constants

Photoelectric Effect

- Differential cross section for K-shell
  - Phys. Rev. 113, 514, 1959
  - β, γ, θ – photoelectron parameters
  - Transverse photoelectron emission

$$\frac{d\sigma}{d\cos\vartheta} \sim \frac{\sin\vartheta^2}{(1-\beta\cos\vartheta)^4}\left[1+\frac{1}{2}\gamma(\gamma-1)(\gamma-2)(1-\beta\cos\vartheta)\right]$$

Coherent (Reyleigh) Scattering

- Differential cross

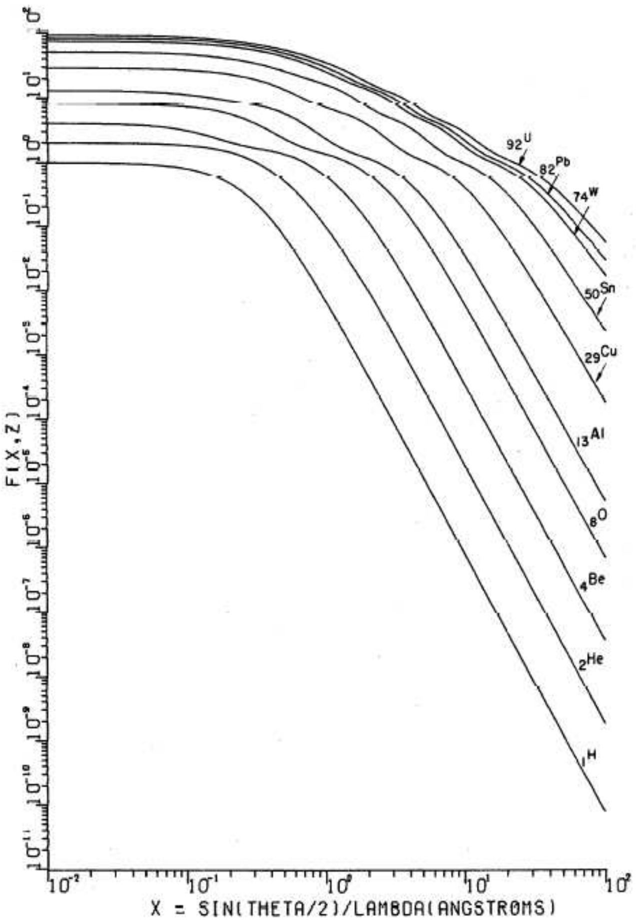
$$\frac{d\sigma}{d(\cos\vartheta)} = \pi r_e^2 \left(1+\cos\vartheta^2\right)\left|F(Z,x)\right|^2, x = \sin(\vartheta/2)/\lambda$$

- For Z > 2 and

$$F(x, Z) = 4\pi \int_0^\infty r^2 \rho(r, Z) \cdot \frac{\sin(4\pi x r)}{4\pi x r}\, dr$$

- Parameterization of atomic form-factors from

J.H.Hubbell, J.Phys.Chem.Ref.Data 8, 69, 1979



Compton Scattering

- The quasi-free scattering $\gamma + e \rightarrow \gamma' + e'$
- Klein-Nishina formula (no polarization):

$$\frac{d\sigma}{dk'} = \frac{\pi r_e^2}{mc^2} \frac{Z}{\kappa^2} \left[\epsilon + \frac{1}{\epsilon} - \frac{2}{\kappa} \left(\frac{1-\epsilon}{\epsilon} \right) + \frac{1}{\kappa^2} \left(\frac{1-\epsilon}{\epsilon} \right)^2 \right]$$

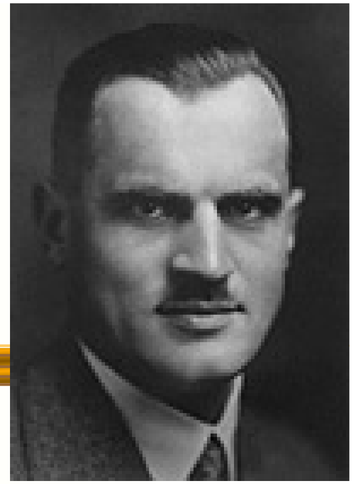
k' energy of the scattered photon ; $\epsilon = k'/k$

r_e classical electron radius

κ k/mc^2

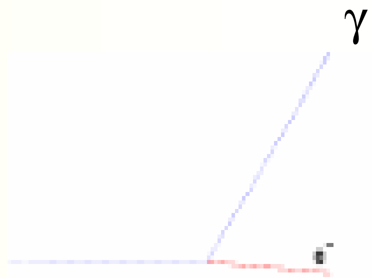
- Low energy limit: $$\frac{d\sigma}{dk'} = \left[\frac{d\sigma}{dk'} \right]_{KN} \times S(k, k')$$
- S(k, k') – scattering function depending on atomic shell structure

Compton Scattering



$$\sigma(k) = \int_{k'_{min}=k/(2\kappa+1)}^{k'_{max}=k} \frac{d\sigma}{dk'}\, dk'$$

Kinematics is defined by final energy of γ



$$\sigma(k) = 2\pi\, r_e^2\, Z \left[\left(\frac{\kappa^2 - 2\kappa - 2}{2\kappa^3} \right) \ln(2\kappa+1) + \frac{\kappa^3 + 9\kappa^2 + 8\kappa + 2}{4\kappa^4 + 4\kappa^3 + \kappa^2} \right]$$

limits

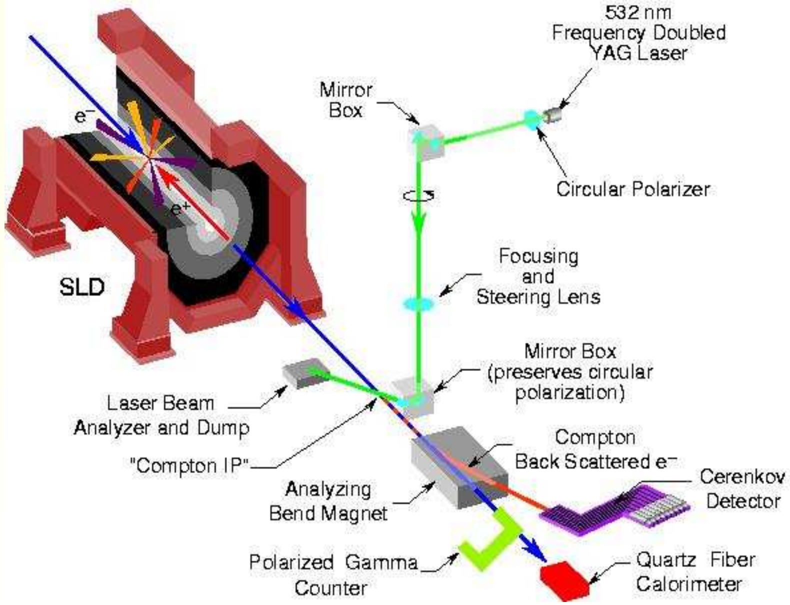
$k \to \infty :$ $\quad \sigma$ goes to 0 : $\sigma(k) \sim \pi\, r_e^2\, Z\, \dfrac{\ln 2\kappa}{\kappa}$

$k \to 0 :$ $\quad \sigma \to \dfrac{8\pi}{3}\, r_e^2\, Z$ (classical Thomson cross section)

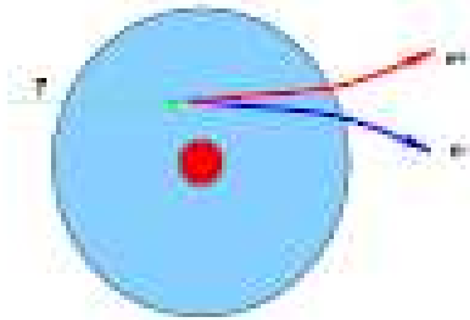
Structure function
Saturate Thomson
Cross section ~k^2

Compton Scattering for Measurement of Electron Beam Polarization at SLC, SLAC

Compton effect in different kinematics



Gamma Conversion



- Born approximation (Bethe-Haitler):

$$\sigma(Z,E_\gamma) = Z(Z+1)\alpha r_e^2 \frac{2\pi}{3}\left(\frac{k-2}{k}\right)^3 F(k), k = \frac{E_\gamma}{mc^2}$$

- Necessary corrections:
  - Coulomb corrections (next after Born orders)
  - The screening of the field of the nucleus
  - Pair creation in the field of atomic electrons
  - LPM effect – the formation length suppression
  - Practical parameterization:

J.H.Hubbell, J.Phys.Chem.Ref.Data 9, 1023, 1980

Gamma Conversion – Differential Cross Section

high energies regime : $E_\gamma \gg m_e c^2/(\alpha Z^{1/3})$

Above few GeV the energy spectrum formula becomes simple :

$$\left.\frac{d\sigma}{d\epsilon}\right]_{Tsai} \approx 4\alpha r_e^2 \times \left\{ \left[1 - \frac{4}{3}\epsilon(1-\epsilon)\right] \left(Z^2 \left[L_{rad} - f(Z)\right] + Z L'_{rad}\right) \right\}$$

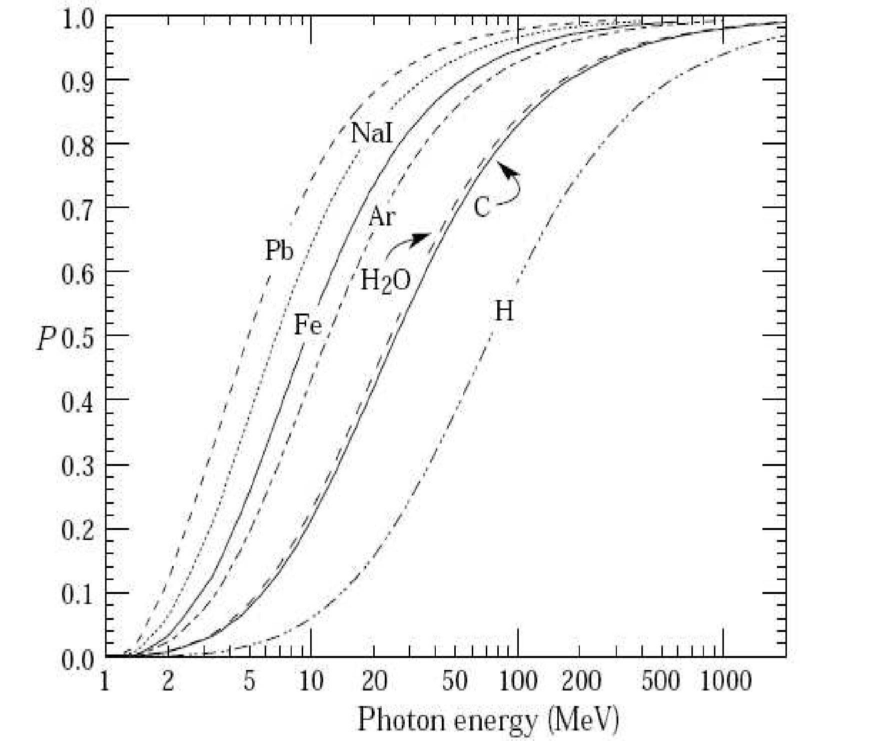
where

E_γ	energy of the incident photon
E	total energy of the created e^+ (or e^-) ; $\epsilon = E/E_\gamma$
$L_{rad}(Z)$	$\ln(184.15/Z^{1/3})$ (for $Z \geq 5$)
$L'_{rad}(Z)$	$\ln(1194/Z^{2/3})$ (for $Z \geq 5$)
$f(Z)$	Coulomb correction function

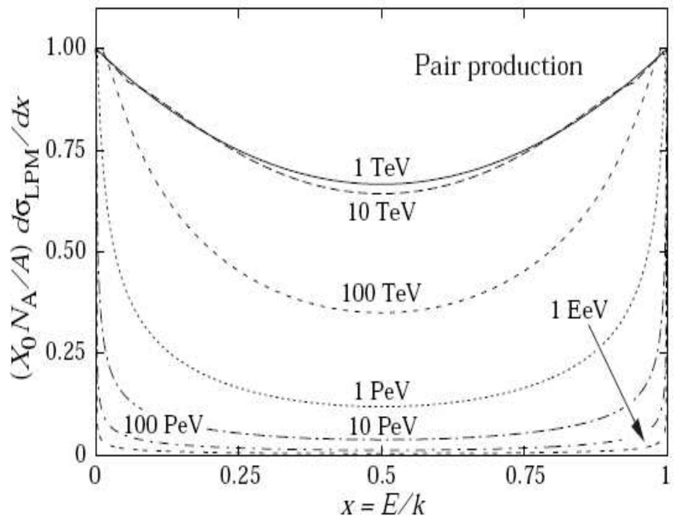
- Bethe-Heitler formula with corrections from
  - *Y.S. Tsai, Rev. Mod. Phys. 46, 815, 1974; 49, 421, 1977*
- The synthesis
  - *S.M. Seitler and M.J.Berger, Int. J. of Appl. Rad. 35, 665, 1984*

Gamma Conversion (PDG plots)

Relative probability of the conversion: At high energies conversion dominates



Asymmetry of electron/positron energies increasing for very high energies of gamma due to LPM effect



Radiation length

- The characteristic distance in a media directly connected with the gamma conversion cross section:

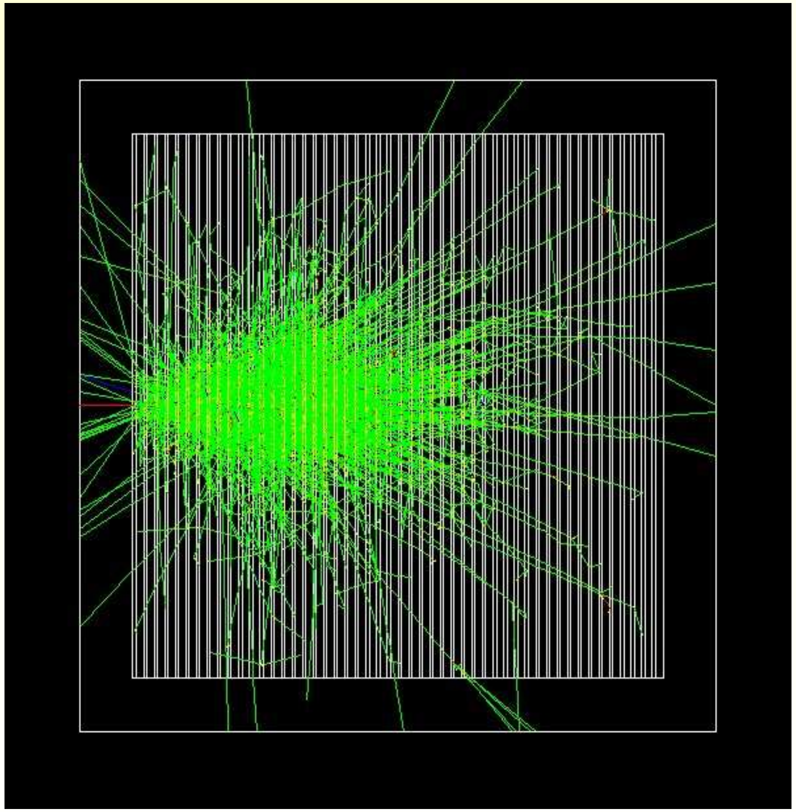
$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 \left[L_{\rm rad} - f(Z) \right] + Z\, L'_{\rm rad} \right\}$$

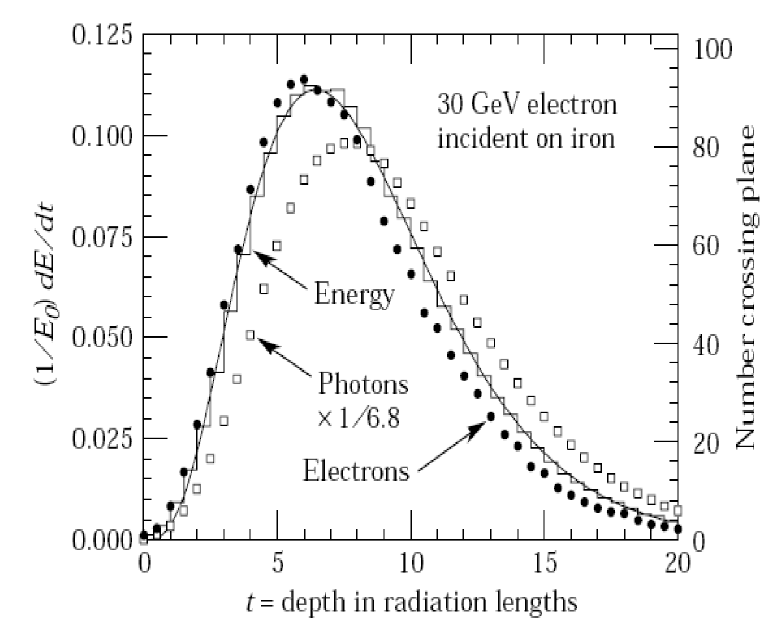
- Approximation

Element	Z	$L_{\rm rad}$	$L'_{\rm rad}$
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

Y.S. Tsai, Rev. Mod. Phys. 46, 81

Longitudinal EM shower profile is scaled in radiation length

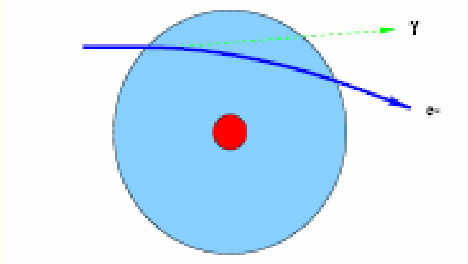




EM shower profile:

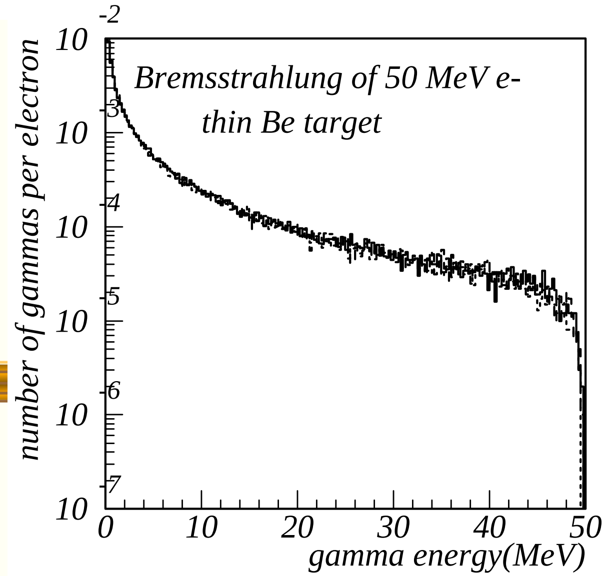
$$\frac{dE}{dt} = E_0\, b \,\frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

Bremsstrahlung



- Inverse process to gamma conversion
- For E > 1 MeV dominate process for e^+ and e^-
- First Born approximation by Bethe-Heitler
- Corrections:
  - Screening of the nucleus field
  - Bremsstrahlung on atomic electrons
  - Next terms after Born
  - Polarization of media (dielectric suppression)
  - LPM – formation length suppression

Bremsstrahlung



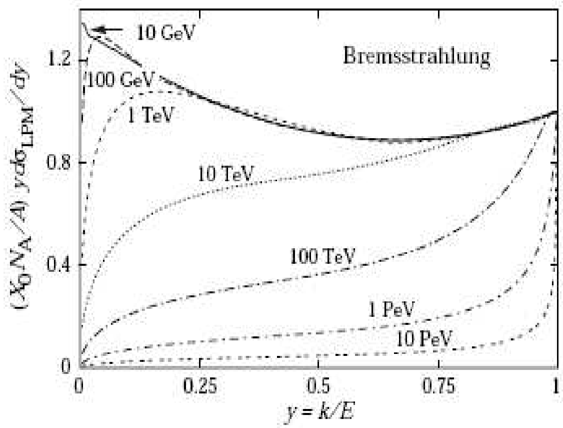
- Differential cross section
  - *Y.S. Tsai, Rev. Mod. Phys. 46, 815, 1974*

$$\frac{d\sigma}{dk} = \frac{4A}{3N_A X_0}\frac{1}{k}\left(1 - y + 0.75y^2\right)S(Z,y),\ y = \frac{k}{E}$$ - assymptotic

- S(Z,y) – dielectric & LPM suppressions

$$S_P = \frac{y^2}{y^2 + \frac{4\pi r_0^3 n_e}{\alpha^2}},$$

$$S_{LPM} = \sqrt{\frac{\alpha m^2 X_0 k}{4hcE^2}},\ \frac{1}{S} = 1 + \frac{1}{S_P} + \frac{S}{S_{LPM}}$$



Bremsstrahlung

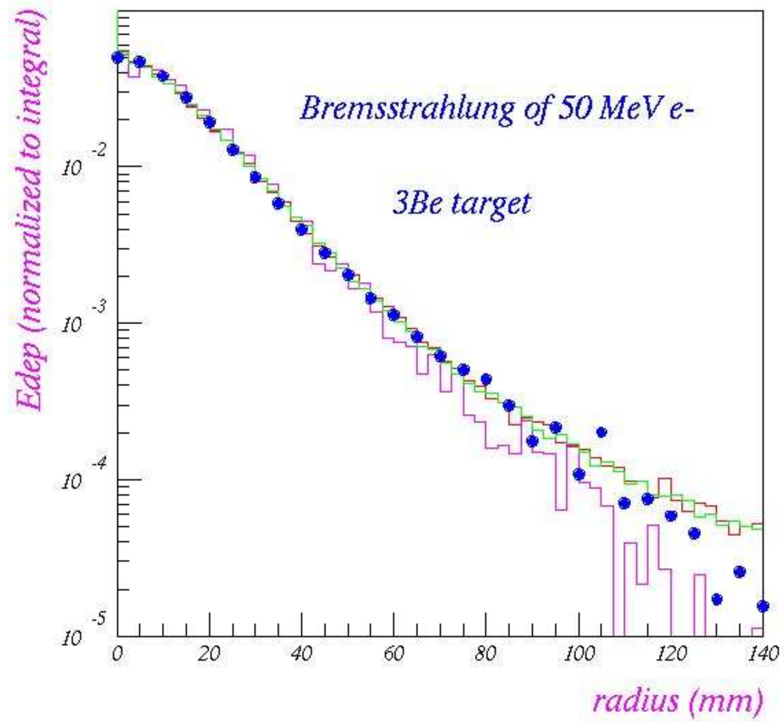
- Tsai approximation of angular distribution:

$$\frac{d\sigma}{dx} = Cx\left(e^{-ax} + 27e^{-3ax}\right),$$

$$a = 0.625, x = \frac{E\vartheta}{m}$$

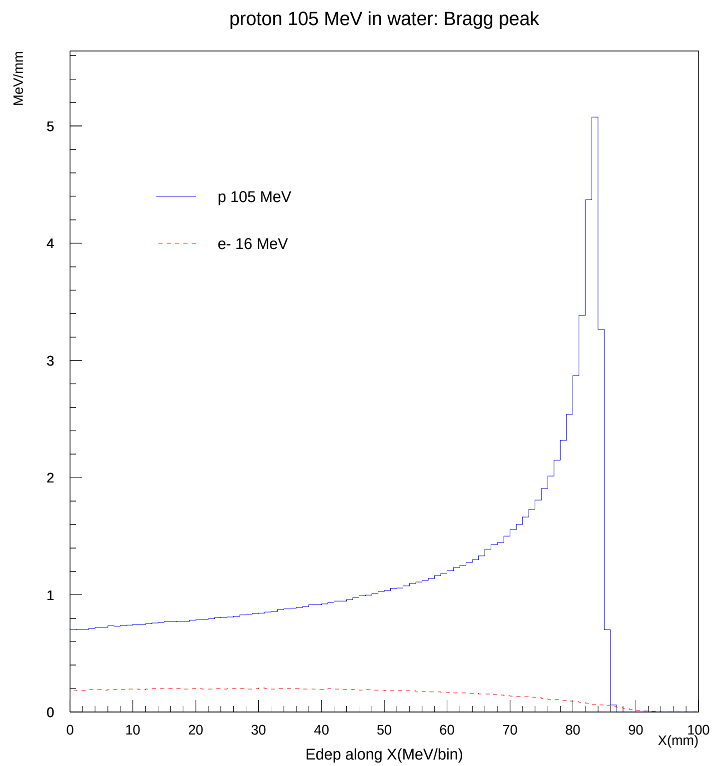
- Is precise for E>1 MeV
- Is not applicable below 100 keV
- Two extra models inside the Lowenergy package

Tsai angular distribution fits the data

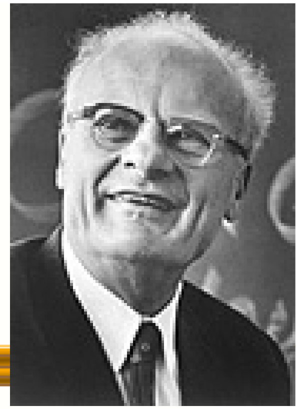


Ionization of Protons and Electrons

- Protons below 1 MeV are highly ionizing and have still enough energy
- Bragg peak of ionization near the end of the heavy particle trajectory
- Parameterization of stopping powers below 1 MeV are done using experimental data
  - A.Allisy, ICRU 49
  - http://physics.nist.gov/PhysRefData



Heavy Particle Ionization



- Corrected Bethe-Bloch formula (A.Allisy, ICRU 49, 1993)

$$-\frac{dE}{dx} = 4\pi N_e r_0^2 \frac{z^2}{\beta^2}\left(\ln\frac{2m_e c^2\beta^2\gamma^2}{I} - \frac{\beta^2}{2}\left(1-\frac{T_c}{T_{\max}}\right) - \frac{C}{Z} + \frac{G-\delta-F}{2} + zL_1 + z^2L_2\right)$$

  - T_c – cut energy ($T_c < T_{max}$)
  - T_{max} – kinematical max energy
  - I – mean ionization potential
  - C – shell correction (increasing for low energies)
  - G – Mott correction (important for ions)
  - δ – density correction (collective media effect)
  - F – finite size correction (important for ions)
  - L_1- Barkas correction (difference in ranges of μ^+ and μ^-)
  - L_2- Bloch correction

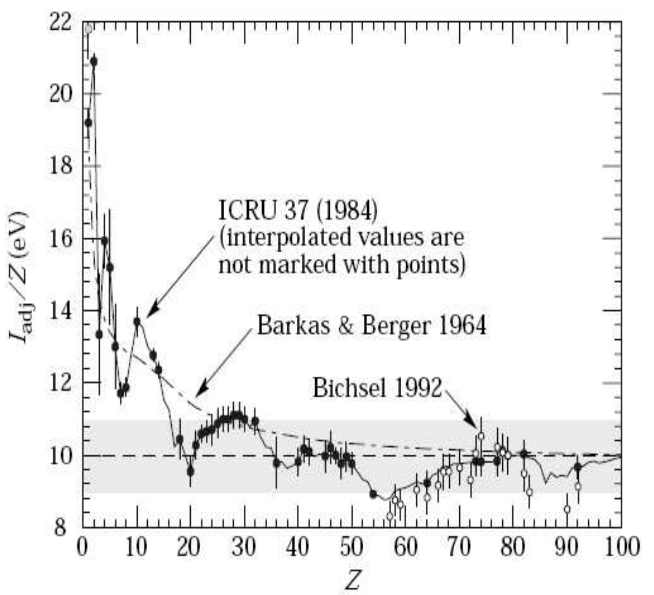
Heavy Particle Ionization Continue

$$T_{\max} = \frac{2m_e c^2 \beta^2\gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

Density effect

$$\delta/2 \to \ln(\hbar\omega_p/I) + \ln\beta\gamma - 1/2$$

$$\delta = \begin{cases} 2(\ln 10)x - \overline{C} & \text{if } x \geq x_1; \\ 2(\ln 10)x - \overline{C} + a(x_1 - x)^k & \text{if } x_0 \leq x < x_1; \\ 0 & \text{if } x < x_0 \text{ (nonconductors)}; \\ \delta_0 10^{2(x-x_0)} & \text{if } x < x_0 \text{ (conductors)} \end{cases}$$



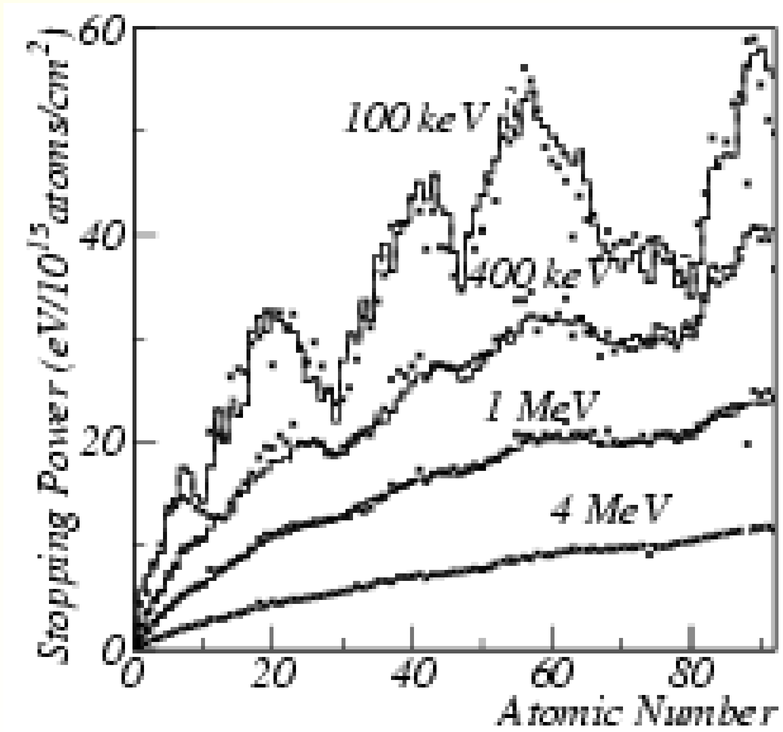
Differential cross section
of δ-electron production
Binding energy is neglected

$$\frac{d\sigma}{dT} \sim \frac{z^2}{\beta^2}\frac{F(T)}{T^2}, F(T)_{S=0} = \left(1 - \beta^2 \frac{T}{T_{\max}}\right)$$

Heavy Particle Ionization Continue

- Energy loss is also called "Stopping Power" (see ICRU)
- For T< 1 MeV strong shell dependence of stopping power
  - Ionization grow up as $1/\beta^2$
  - Max when atomic electrons velocity is about particle velocity
  - Screening effect saturate energy loss

Proton stopping power



Electron/Positron Ionization

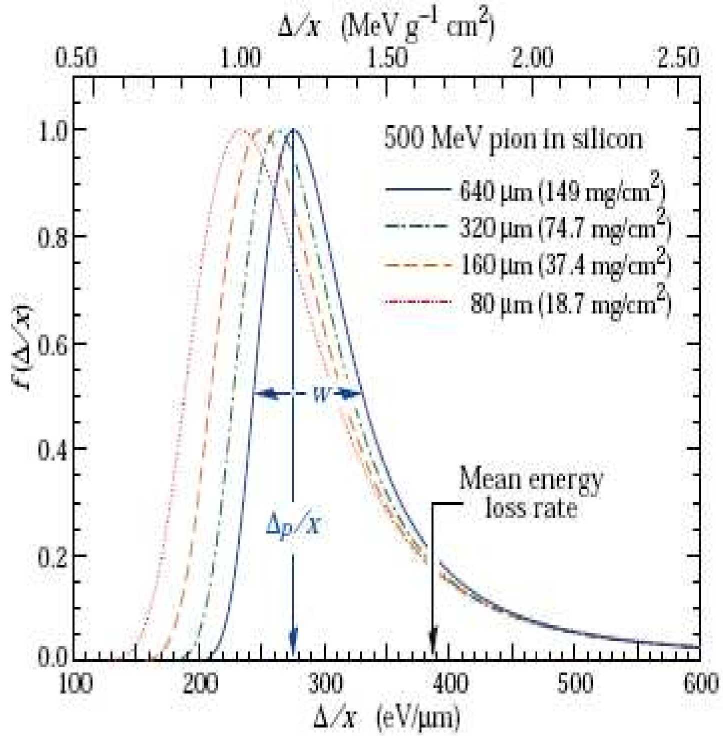
H. Messel and D.F. Crowford, Pergamon Press, Oxford, 1970

$$\left.\frac{dE}{dx}\right]_{T<T_{cut}} = 2\pi r_e^2 mc^2 n_{el} \frac{1}{\beta^2}\left[\ln\frac{2(\gamma+1)}{(I/mc^2)^2} + F^{\pm}(\tau,\tau_{up}) - \delta\right]$$

r_e	classical electron radius: $e^2/(4\pi\epsilon_0 mc^2)$
mc^2	mass energy of the electron
n_{el}	electron density in the material
I	mean excitation energy in the material
γ	E/mc^2
β^2	$1-(1/\gamma^2)$
τ	$\gamma - 1$
T_{cut}	minimum energy cut for δ -ray production
τ_c	T_{cut}/mc^2
τ_{max}	maximum energy transfer: τ for e^+, $\tau/2$ for e^-
τ_{up}	$\min(\tau_c, \tau_{max})$
δ	density effect function.

Fluctuations of Energy Loss

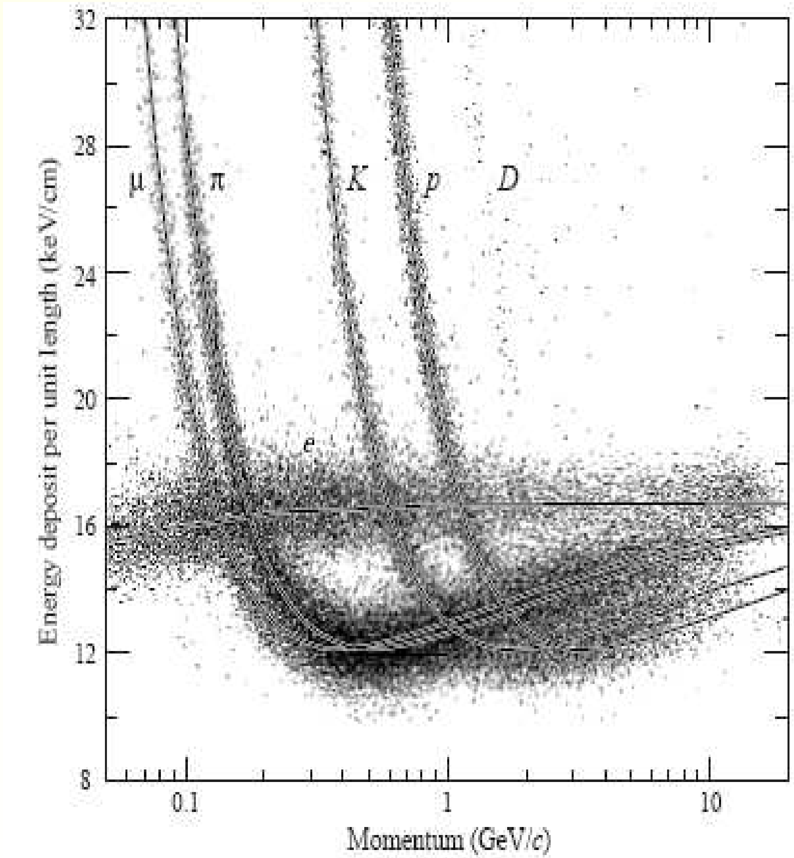
- In "thin" absorbers mean energy loss and the most probable energy loss are different significantly
- In "thick" absorbers, when energy loss about kinetic energy the distribution is Gaussian
- Fluctuations in energy loss provides struggling of particle range



H.Bichsel, Rev. Mod. Phys., 60, 663, 1988

Particle Identification

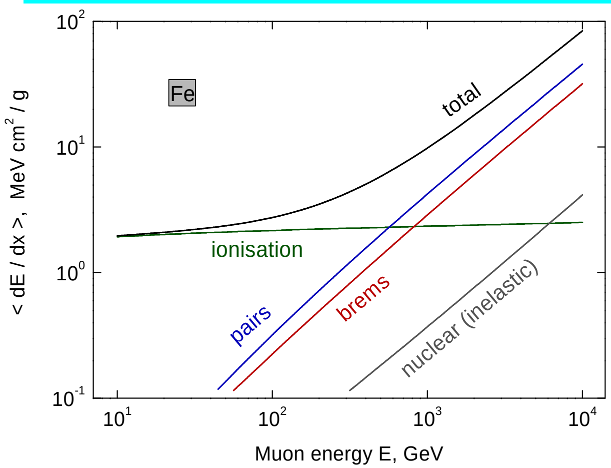
- Energy loss of heavy charged particles are function of β
- In magnetic spectrometers particle momentum and sign of its charge can be measured
- Combine with the dE/dx measurement in gases for identification



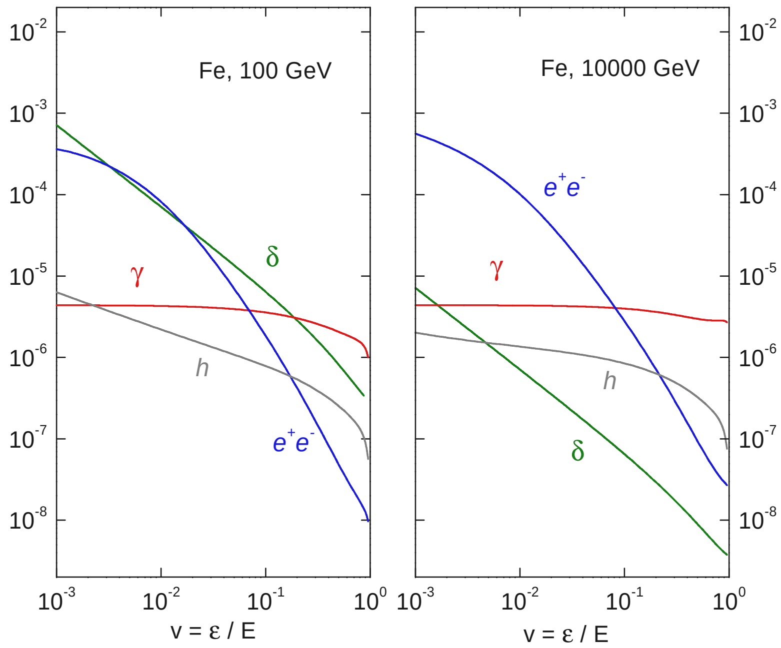
Muon Interactions with Matter

- Basic processes:
  - Ionisation
  - Bremsstrahlung
  - Production of e^+e-
  - Muon-nuclear interactio

Total muon energy loss



Muon differential cross sections

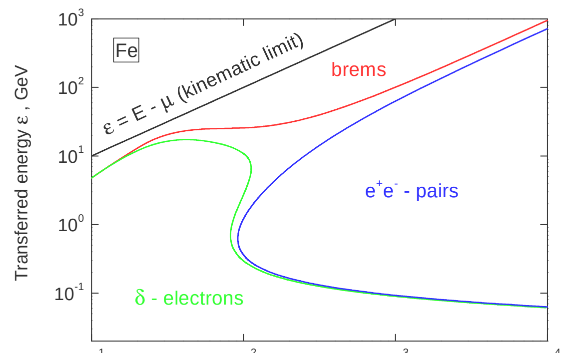


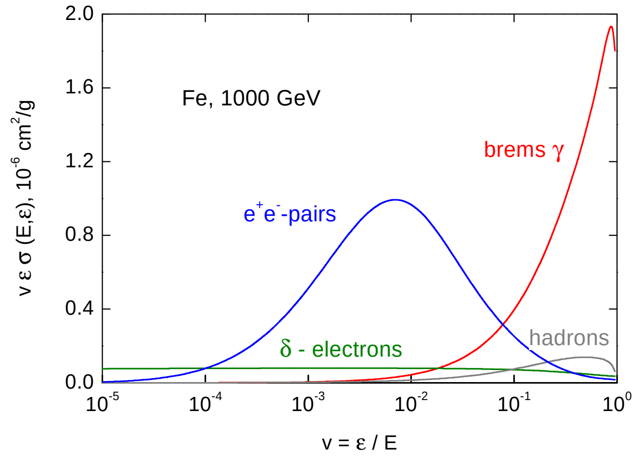
Courtesy R.Kokoulin

Muon interactions

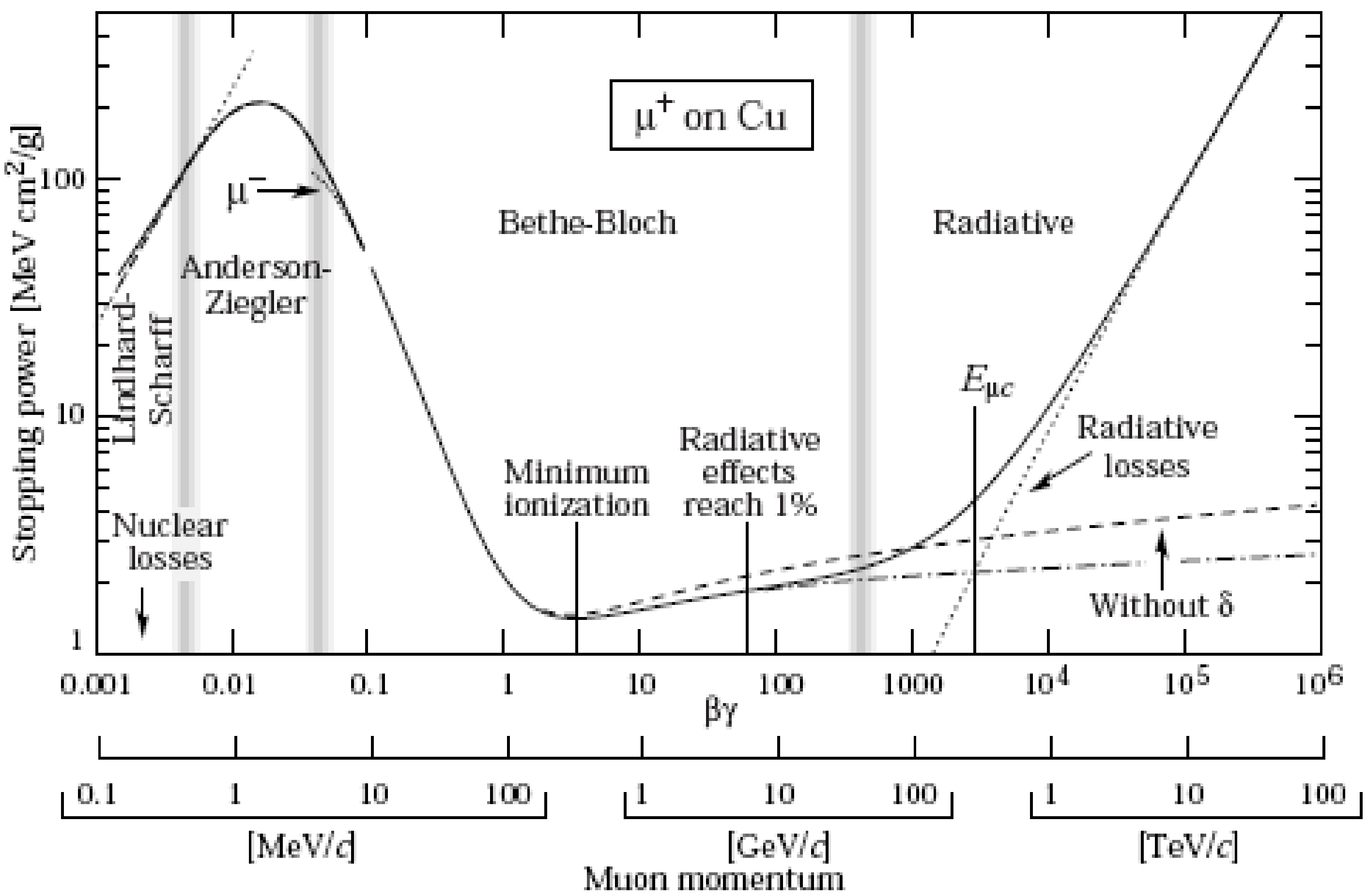
Muon cross sections: curves indicate areas, in which a process give >50% contribution

- ✶ At moderate energies - δ-electrons
- ✶ For high energy e^+e^- production dominates
- ✶ At highest transfers ε/E > 0.1 dominates bremsstrahlung (catastrophic energy loss)

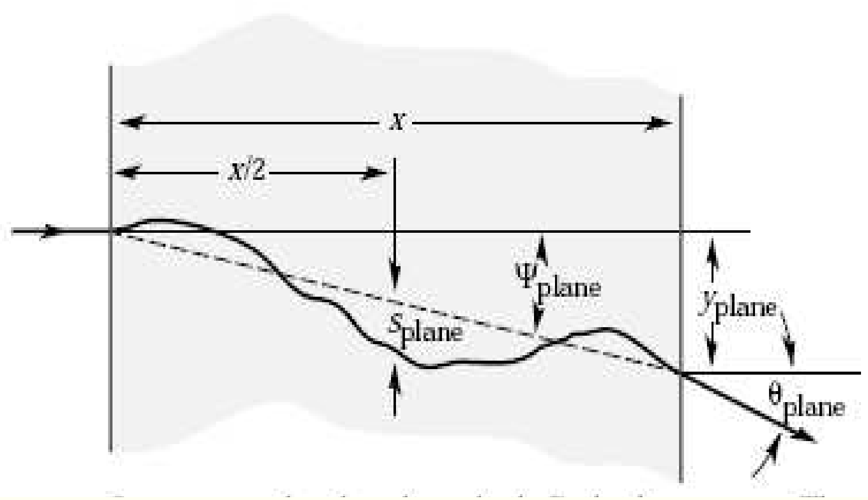




Muon Stopping Power (PDG)



Multiple Scattering



$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p} z \sqrt{x/X_0}\Big[1 + 0.038 \ln(x/X_0)\Big]$$

V.L.Highland, NIM 129, 497, 1975

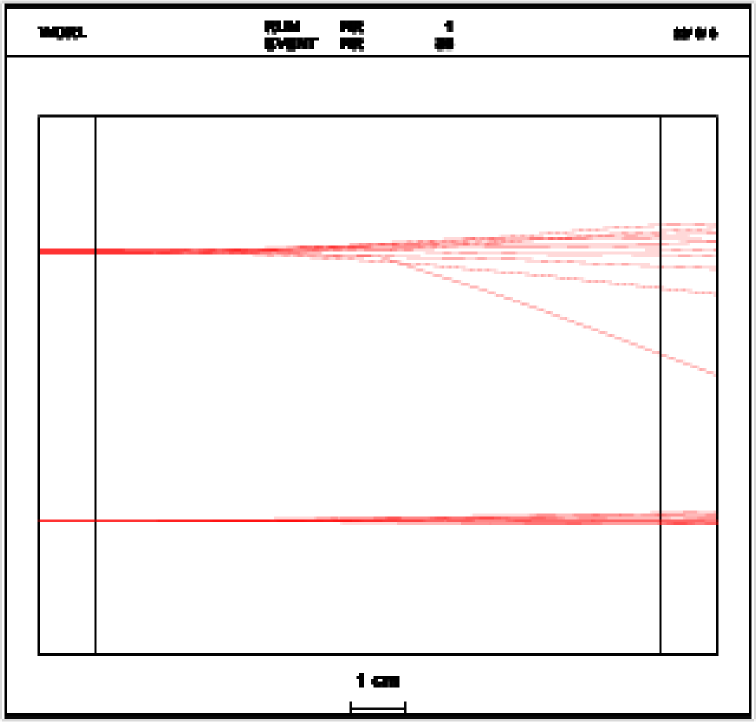
Is accurate for $10^{-3} < x/X_0 < 100$

- Is applied if particle does many soft collisions with atomic electrons and nuclei
- This is an approximation for small angles with accuracy ~10%
- Hard Reserford collisions are not taken into account

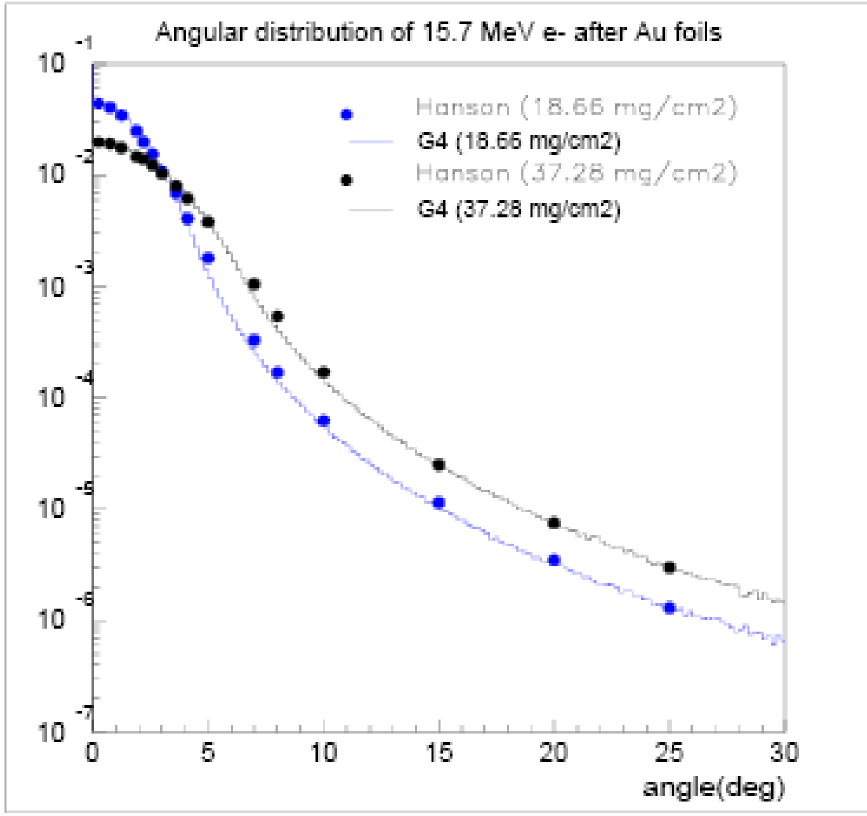
Multiple Scattering Illustrations

Energy dependence

10 π^+ of 200 MeV and 1 GeV crossing 10 cm of Aluminium.

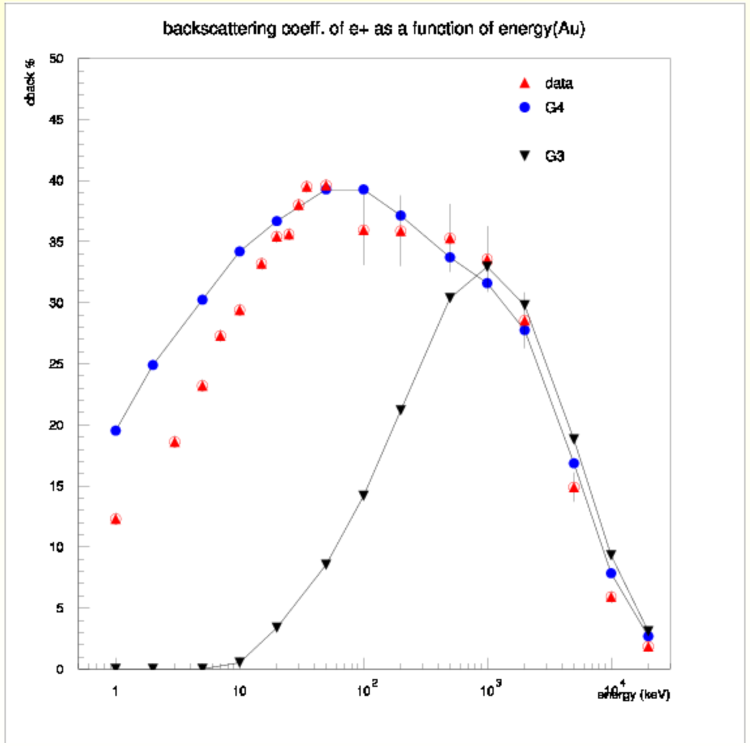


Angle distributions central part + tail

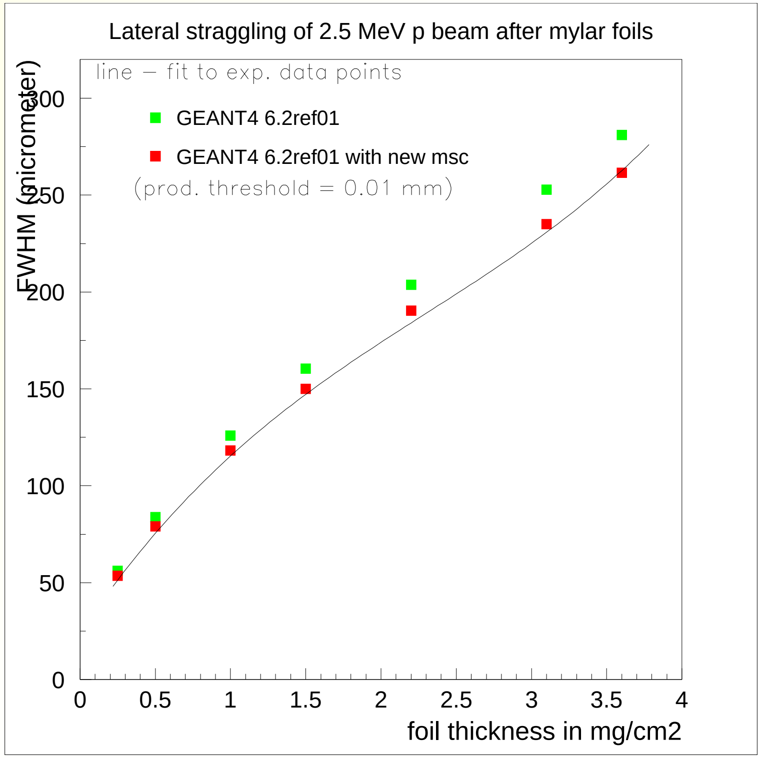


Multiple scattering model of L.Urban

Back scattering

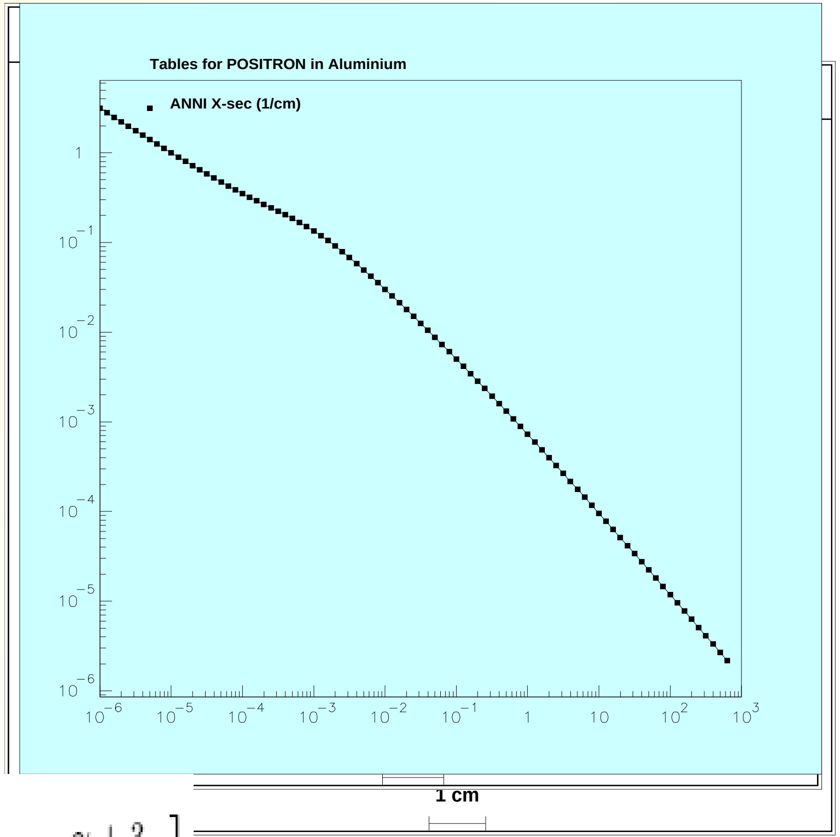


Transverse displacement



Positron Annihilation

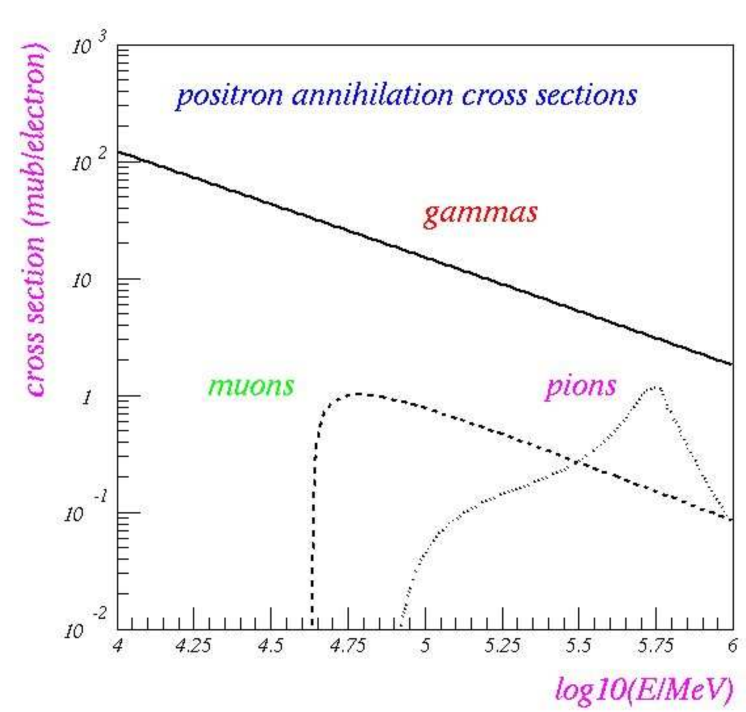
- Takes place on-fly and at rest
- Monochromatic gammas in rest frame
- Key process for positron tomography
- Cross section:



$$\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

High energy EM processes

- EM background due to high energy EM interaction with media:
  - $\gamma \rightarrow \mu^+\mu^-$ ($\sigma \sim Z^2$)
  - $e^+ \rightarrow \mu^+\mu^-$ ($\sigma \sim Z$)
  - $e^+ \rightarrow \pi^+\pi^-$ ($\sigma \sim Z$)
- Visible at LEP and High at SLC
- Of concern for linear colliders



Conclusion remarks

- “Classical” quantum mechanics, its relativistic extension, and the theory of atom allow to describe particle interactions with matter
- Geant4 offer a complete set of physics processes and models for simulation of EM physics
- Standard packages more oriented to HEP but applicable for medical and other applications
- Lowenergy package provides alterative models allowing simulation of atomic effects